

Dynamical Instability of Self-Tuning Solution with Antisymmetric Tensor Field

by

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ABSTRACT

We consider the dynamical stability of a static brane model that incorporates a three-index antisymmetric tensor field and has recently been proposed [1, 2] as a possible solution to the cosmological constant problem. Ultimately, we are able to establish the existence of time-dependent, purely gravitational perturbations. As a consequence, the static solution of interest is “dangerously” located at an unstable saddle point. This outcome is suggestive of a hidden fine tuning in what is an otherwise self-tuning model.

1 Introduction

One of the most serious puzzles of fundamental physics is known as the “cosmological constant problem” [3]. The essence of this problem can be summarized as follows. Recent cosmological observations [4] place the cosmological constant, or effective vacuum energy density, at a very small value in comparison to the Planck scale; in fact, $\Lambda_{obs} \approx 10^{-120} M_{PL}^4$. Conversely to this observational bound, one would naively expect quantum fluctuations in the vacuum energy to be on the order of the Planck scale; i.e., $\Lambda_{theo} \approx M_{PL}^4$. Hence, the cosmological constant problem translates to a hierarchical problem that requires a formidable fine tuning of 120 orders of magnitude.¹

Many interesting, varied attempts have been invoked to resolve this cosmological constant hierarchy; albeit, with only limited degrees of success. Some of these have been based on, for instance, “quintessence” [5], the “anthropic principle” [6] and a probabilistic interpretation of the universe [7]. More recently, a program that makes use of the “brane world” scenario has been applied in this context. We will focus on this approach below.

There currently exists an abundance of different versions and interpretations of the brane world [8]; however, the basic picture is fundamentally consistent. This being that “ordinary” matter is trapped on a 3+1-dimensional submanifold (or “three brane”), whereas the graviton and (possibly) other hypothetical fields can propagate in a 3+1+ n -dimensional bulk. Note that the “extra” bulk dimensions are typically, but not restrictively, compact. The current popularity of the brane world scenario can be traced to its associations with M-theory (of which brane worlds can arise as a low-energy limit [9]), as well as its potential resolution of various hierarchical problems (such as that between the Planck and electroweak scales [10]).

In the context of the cosmological constant problem, it is useful to elaborate on a specific brane theory; namely, the second one proposed by Randall and Sundrum [11], or RS2 as it is commonly known. In this model, a single positive-tension brane is coupled to anti-de Sitter gravity in a 5-dimensional bulk. Note that, for this model, there are no other bulk fields (besides grav-

¹The situation can be somewhat improved if we assume that supersymmetry (or any symmetry which conspires to impose a vanishing vacuum energy) remains unbroken at energies that are just above the present-day accelerator limits. If this were the case, quantum fluctuations could be as small as $10^{60} M_{PL}^4$. However, there would remain at least 60 orders of magnitude to still be explained away.

ity) and the extra dimension is taken to be infinite. RS2 can lead to solutions for which the 4-dimensional (effective) cosmological constant is vanishing. However, such a solution necessitates a fine tuning between the brane tension (V) and the bulk cosmological constant (Λ) such that $V = \sqrt{-12M^3\Lambda}$. (Here, M is the fundamental mass scale in five dimensions.)

It has been hoped that the inclusion of new bulk fields in the RS2 scenario could somehow lead to a model that permits “self-tuning” solutions. That is, a model for which the brane tension could take on an extended range of values without jeopardizing the stability of the 4-dimensional cosmological constant. In this way, such a model would be stable against any radiative corrections to the brane tension and, hence, a state of 4-dimensional Poincare invariance would be preserved.²

One such candidate for a self-tuning model has been proposed by a pair of groups: Arkani-Hamed, Dimopoulos, Kaloper and Sundrum [13], and Kachru, Schulz and Silverstein [14]. The ADKS-KSS model is essentially RS2 with a vanishing bulk cosmological constant and with coupling to a scalar (dilaton) field. Both studies identified apparent self-tuning solutions; and, for certain choices of brane-dilaton coupling, it was also found that curved-brane solutions are conveniently forbidden. However, subsequent studies [15, 16] revealed that the naked singularities, which are inherent in these solutions,³ lead to inconsistencies in the 4-dimensional effective field theory. The resolution of these inconsistencies necessitates that an additional brane be added at each singular point. As each new brane leads to additional boundary conditions, it is not surprising that 4-dimensional Poincare invariance can only be achieved via (at least one) fine tuning.

As demonstrated in Ref.[17], the failure of the ADKS-KSS model to support (legitimate) self-tuning solutions is really just a generic feature of a wide class of brane models with coupling to a bulk scalar. Furthermore, the situation does not appear to be rectified when higher-order curvature terms are included [18].

Even with the issue of naked singularities put aside, the ADKS-KSS model has been critiqued on other grounds. Binetruy et al. [20] have demonstrated

²Many models that support flat-brane solutions have been shown to support curved-brane solutions as well [12]. However, one can assume that the flat-brane solutions are significantly more favorable by invoking probabilistic [7] or anthropic [6] principles.

³Actually, these singularities can be avoided; but only along with the unphysical consequence of $M_{PL} \rightarrow \infty$, where M_{PL} is the 4-dimensional effective Planck mass.

that the field equations (including the “jump” conditions [19]) support time-dependent perturbed modes that do **not** jeopardize the Poincare invariance on the brane.⁴ Such perturbations imply that the static “self-tuned” solutions are dynamically unstable. From this outcome, it can be further inferred that the brane world must evolve either from or into a singularity, the 4-dimensional Planck mass is time dependent, and energy fails to be conserved on the brane. This lack of stability has also been substantiated by Diemand et al. [21] via an alternative (but related) approach. Furthermore, the latter study found analogous instabilities arising in a self-tuning model that has been proposed by Kehagias and Tamvakis [22].⁵

Another promising candidate for a self-tuning theory has been recently documented by Kim, Kyae and Lee [1, 2]. Similarly to the ADKS-KSS case, the KKL model is essentially the RS2 scenario along with a new bulk field. However, rather than a scalar field, Kim et al. have proposed a three-index antisymmetric tensor field. Just such a field has natural origins under the compactification of 11-dimensional supergravity [23] and has previously been considered, as far back as 1980 [24, 25], in the context of the cosmological constant problem.

One might anticipate that a three-index antisymmetric tensor gives rise to an action term of the form H^2 [7] (where H represents the four-index field-strength tensor). Conversely to such intuition, the KKL model rather contains an unorthodox H^{-2} term. It has been shown [2], however, that a negative power of H^2 is critical to the self-tuning properties of the model. For such a term to make sense, H^2 must develop a vacuum expectation value on the order of the fundamental mass scale. In this way, the KKL action could perhaps represent an effective theory that arises out of quantum gravity.

Interestingly, the KKL model allows for a static self-tuning solution that is endowed with both a finite 4-dimensional Planck mass and an absence of singularities throughout the bulk. It remains an open question, however, whether or not the KKL model suffers from dynamical instabilities of the type

⁴Perturbations of this type (i.e., for which the brane remains flat) should not be confused with perturbed modes that induce a finite curvature on the brane. As previously noted, curved-brane solutions may be probabilistically suppressed.

⁵The KT model [22] consists of 5-dimensional gravity and a bulk scalar field, but no brane. Rather, for a specific choice of dilaton potential, there exists a parameter limit by which a flat brane is effectively realized. Even in this limiting case, the solutions are notably free of any singularities. However, the dynamics of the brane limit remain unclear.

that plague the ADKS-KSS and KT models (as discussed above) [20, 21]. The purpose of the current paper is to rigorously address this issue.

The remainder of this paper proceeds as follows. In Section 2, we present the KKL action and corresponding field equations, after which the static solution is discussed. This section can be regarded as a review of Ref.s[1, 2]. In Section 3, we consider a time-dependent analysis of the field equations. Applying a methodology that has been inspired by Ref.[21], we are indeed able to verify the existence of stability-threatening perturbed modes. Section 4 ends with a brief summary and discussion.

2 Field Equations and Static Solution

We begin the formal analysis by recalling the KKL action as introduced in Ref.[1]. This action describes gravity and a three-index antisymmetric tensor field A_{MNP} existing in five dimensions and coupled to a 4-dimensional domain wall or brane (which can be positioned at $y = 0$, where y is the “extra” bulk dimension, without loss of generality). More specifically, the action of interest can be expressed as:

$$S = \int dx^4 \int dy \sqrt{-g} \left[\frac{1}{2\kappa^2} R + \frac{2 \cdot 4!}{H^2} - \Lambda - V\delta(y) \right], \quad (1)$$

where $\kappa^{-2} = 2M^3$ (with M being the fundamental mass scale) and where $H^2 = H_{MNPQ}H^{MNPQ}$ is the square of the field strength for which $H_{MNPQ} = \partial_{[M}A_{NPQ]}$. (For a brief discussion on this choice of H^{-2} , see Section 1.) Note that the bulk cosmological constant Λ and the brane tension V are assumed to be a negative and positive constant, respectively. In principle, Λ is determined by some higher-dimensional, fundamental theory, whereas V contains all the information regarding the Standard Model physics that lives on the brane.

Varying the action with respect to the metric and antisymmetric tensor field, we obtain the following field equations:

$$\frac{G_{MN}}{\kappa^2} = -g_{MN}\Lambda - g_{\mu\nu}\delta_M^\mu\delta_N^\nu V\delta(y) + 2 \cdot 4! \left(\frac{8}{H^4} H_{MPQR}H_N{}^{PQR} + g_{MN}\frac{1}{H^2} \right), \quad (2)$$

$$\partial_M \left(\sqrt{-g} \frac{H^{MNPQ}}{H^4} \right) = 0, \quad (3)$$

where G_{MN} is the Einstein tensor and Greek indices represent brane coordinates.

Having interest in the dynamical behavior of solutions with 4-dimensional Poincare invariance, we invoke the following ansatz for the metric:

$$ds^2 = e^{2A(t,y)} \eta_{\mu\nu} dx^\mu dx^\nu + b^2(t,y) dy^2. \quad (4)$$

On the basis of earlier studies [7, 23, 24, 25], one can anticipate that the four-index field-strength tensor is expressible in terms of a single massless scalar field and a purely geometrical, antisymmetric tensor field. We thus impose the following ansatz (first suggested in Ref.[2]) on the field strength:⁶

$$H^{\mu\nu\rho\eta} = \frac{\epsilon^{\mu\nu\rho\eta}}{\sqrt{-g}} \frac{\partial}{\partial y} \sigma(y, t), \quad (5)$$

$$H^{4ijk} = \frac{\epsilon^{4ijk}}{\sqrt{-g}} \frac{\partial}{\partial t} \sigma(y, t), \quad (6)$$

$$H^{04\mu\nu} = 0, \quad (7)$$

where ϵ^{MNPQ} is the four-index (contravariant) Levi-Civita symbol, Roman indices represent spatial brane coordinates, the index 0/4 represents the coordinate t/y and all permutations have been implied.

Next, we re-express the field equations (2,3) in terms of the complete ansatz (4-7). Straightforward but tedious calculations yield the following equations in the bulk:

$$3e^{-2A} \left(\dot{A}^2 b^2 + \dot{A} \dot{b} b \right) - 6A'^2 - 3A'' + 3 \frac{A'b'}{b} = (\kappa b)^2 \left[\Lambda + \frac{6}{f^2 - h^2} + \frac{4h^2}{(f^2 - h^2)^2} \right], \quad (8)$$

$$3e^{-2A} \left(2\ddot{A} b^2 + \dot{A}^2 b^2 + \dot{A} \dot{b} b + \ddot{b} b \right) - 6A'^2 - 3A'' + 3 \frac{A'b'}{b} = (\kappa b)^2 \left[\Lambda + \frac{6}{f^2 - h^2} \right], \quad (9)$$

$$3b^2 e^{-2A} \left(\ddot{A} + \dot{A}^2 \right) - 6A'^2 = (\kappa b)^2 \left[\Lambda + \frac{2}{f^2 - h^2} - \frac{4h^2}{(f^2 - h^2)^2} \right], \quad (10)$$

$$-3\dot{A} + 3A' \frac{\dot{b}}{b} = 4\kappa^2 \frac{\sigma' \dot{\sigma}}{(f^2 - h^2)^2}, \quad (11)$$

⁶Notably, the four-index field-strength tensor provides a natural way of separating a 3+1-dimensional spacetime out of the 5-dimensional bulk [23].

$$\partial_t \left[\frac{\sigma'}{(f^2 - h^2)^2} \right] - \partial_y \left[\frac{\dot{\sigma}}{(f^2 - h^2)^2} \right] = 0, \quad (12)$$

where $f = b^{-1}\sigma'$, $h = e^{-A}\dot{\sigma}$ and prime/dot denotes differentiation with respect to y/t . Note that Eqs.(8-11) correspond to the 00, jj , 44 and 04 components of the Einstein equation, while Eq.(12) is that obtained by varying the antisymmetric tensor field.

So far, we have neglected the delta-function term in Eq.(2). This term leads to a discontinuity in the “warp” function $A(t, y)$ at the brane. Consequently, we find that the following “jump” condition [19] must be satisfied:

$$A'(y = 0^+) = -\kappa^2 \frac{V}{6} b(y = 0^+). \quad (13)$$

To obtain this form, Z_2 (i.e., reflection) symmetry has been assumed.

Before proceeding with the dynamical analysis, we briefly summarize the results of Ref.s[1, 2] for a static solution with $b(y, t) = 1$. In this case, the field equations (8-12) and boundary condition (13) reduce to the following:

$$6A_o'^2 + 3A_o'' = -\kappa^2 \left(\Lambda + \frac{6}{f_o^2} \right), \quad (14)$$

$$6A_o'^2 = -\kappa^2 \left(\Lambda + \frac{2}{f_o^2} \right), \quad (15)$$

$$A_o'(y = 0^+) = -\kappa^2 \frac{V}{6}, \quad (16)$$

where $f_o = \sigma'_o$ and the subscript o is used to denote the static solution.

Given Z_2 symmetry, the above equations can be uniquely solved (up to a pair of integration constants) to yield:

$$A_o(y) = -\frac{1}{4} \ln \left[\left(\frac{a}{k} \right) \cosh(4k|y| + c) \right], \quad (17)$$

$$f_o = \sigma'_o(y) = \frac{\kappa}{\sqrt{3}k} \cosh(4k|y| + c), \quad (18)$$

where $k = \kappa\sqrt{-\Lambda/6}$, while a and c are the constants of integration. a need only be restricted to having a positive value (and can be determined in terms

of the 4-dimensional effective Planck mass), whereas c can be fixed via the jump condition (16) as follows:

$$\tanh(c) = \kappa \frac{V}{\sqrt{-6\Lambda}}. \quad (19)$$

This relation leads to the restriction $\kappa^2 V^2 < -6\Lambda$, but the brane tension V is otherwise free to adopt any positive value.

From an inspection of the static solution, the desirable features of the KKL model are clearly evident. The integration constant c can adjust itself to moderate changes in the external parameters V and Λ (such as quantum corrections), thereby “protecting” the 4-dimensional Poincare invariance. Hence, the static solution can be classified as one of self tuning. Moreover, the KKL solution has no singularities while still generating a finite value for the 4-dimensional (effective) Planck mass.⁷

3 Linearizing the Field Equations

To examine the time-dependent behavior of this model, it is first convenient to linearize the relevant equations about the static solution. Following Die-
mand et al. [21], we now express the metric functions and scalar σ as follows:

$$A(t, y) = A_o(y) + \delta A(t, y), \quad (20)$$

$$b(t, y) = 1 + \delta b(t, y), \quad (21)$$

$$\sigma(t, y) = \sigma_o(y) + \delta \sigma(t, y). \quad (22)$$

To first order in “ δ ”, the bulk field equations (8-12) take on the following form:

$$12A'_o \delta A' + 3\delta A'' - 3A'_o \delta b' = -\kappa^2 \left[2\delta b \left(\Lambda + \frac{12}{f_o^2} \right) - \frac{12}{f_o^3} \delta \sigma' \right], \quad (23)$$

$$e^{-2A_o} (2\ddot{\delta A} + \ddot{\delta b}) = 0, \quad (24)$$

⁷The Planck mass M_{PL} can be directly evaluated via $M_{PL}^2 = 2M^3 \int_0^\infty \exp[2A_o(y)] dy$. This has been shown to be a finite quantity [2], as long as $\sqrt{k/a}$ is finite.

$$-3e^{-2A_o}\ddot{\delta}A + 12A'_o\delta A' = -\kappa^2 \left[2\delta b \left(\Lambda + \frac{4}{f_o^2} \right) - \frac{4}{f_o^3}\delta\sigma' \right], \quad (25)$$

$$3\dot{\delta}A' - 3A'_o\dot{\delta}b = -\frac{4\kappa^2}{f_o^3}\delta\dot{\sigma}, \quad (26)$$

$$\dot{\delta}b - \frac{1}{f_o}\dot{\delta}\sigma' + \frac{f'_o}{f_o^2}\dot{\delta}\sigma = 0. \quad (27)$$

Note that Eq.(24) corresponds to the difference between Eq.(9) and Eq.(8). The first-order jump condition (13) can now be expressed as:

$$\delta A'(y = 0^+) = A'_o(y = 0^+)\delta b(y = 0^+), \quad (28)$$

where we have also made use of Eq.(16).

For the sake of simplicity, let us now assume that the time-dependent perturbations are linear in t . Hence, Eq.(24) and the first term in Eq.(25) can be disregarded.

It is convenient to define the following combinations:

$$\Psi \equiv \delta A' - A'_o\delta b + \frac{4}{3}\kappa^2\frac{\delta\sigma}{f_o^3}, \quad (29)$$

$$\Phi \equiv \delta b - \frac{\delta\sigma'}{f_o} + \frac{f'_o}{f_o^2}\delta\sigma. \quad (30)$$

With these definitions and the following useful result (cf. Eqs.(14,15)):

$$A''_o = -\frac{4}{3}\kappa^2 f_o^{-2}, \quad (31)$$

the first-order bulk equations (23,25-27) can be rewritten as follows:

$$4A'_o\Psi + \Psi' - 4A''_o\Phi = -4\frac{A'_oA''_o}{f_o}\delta\sigma - \frac{A''_of'_o}{f_o^2}\delta\sigma, \quad (32)$$

$$4A'_o\Psi - A''_o\Phi = -4\frac{A'_oA''_o}{f_o}\delta\sigma - \frac{A''_of'_o}{f_o^2}\delta\sigma, \quad (33)$$

$$\dot{\Psi} = 0, \quad (34)$$

$$\dot{\Phi} = 0. \quad (35)$$

By taking the time derivatives of Eq.(32) and Eq.(33), we are able to deduce that at least one of $\dot{\delta}\sigma = 0$ and:

$$A'_o = -\frac{1}{4}\frac{f'_o}{f_o} \quad (36)$$

must be valid. Regardless of the former, the latter can be readily verified by way of the static solution (17,18). Hence, the first-order bulk equations reduce to the simplified form:

$$4A'_o\Psi + \Psi' = 4A''_o\Phi, \quad (37)$$

$$4A'_o\Psi = A''_o\Phi, \quad (38)$$

$$\dot{\Psi} = \dot{\Phi} = 0. \quad (39)$$

As an important aside, we point out that the combinations Ψ and Φ possess a special property. Namely, if we consider “physical” diffeomorphisms (i.e., those for which the background metric is invariant), then Ψ and Φ can be shown to be invariant under such transformations. To explicitly demonstrate this property, we first note that an infinitesimal diffeomorphism of this type can be described by a Lie derivative with respect to some vector X^M . The first-order perturbations are then expected to transform as follows [21]:

$$\delta A \rightarrow \delta A + A'_o X^4, \quad (40)$$

$$\delta b \rightarrow \delta b + (X^4)', \quad (41)$$

$$\delta\sigma \rightarrow \delta\sigma + \sigma'_o X^4. \quad (42)$$

By virtue of these relations and Eqs.(29-31), it is now evident that $\Psi \rightarrow \Psi$ and $\Phi \rightarrow \Phi$. The significance of this invariant behavior is as follows. If the first-order field equations (37-39) give rise to non-vanishing perturbations, these modes can **NOT** be locally transformed away by a physical diffeomorphism.⁸

It is a straightforward process to solve Eqs.(37-39), which yields:

$$\Psi = C e^{12A_o}, \quad (43)$$

⁸That is, we are justified in replacing δb , δA and $\delta\sigma$ with their gauge-invariant forms. These forms are explicitly given in Ref.[21] and henceforth implied.

$$\Phi = 4 \frac{A'_o}{A''_o} C e^{12A_o}, \quad (44)$$

where C is some constant. We can now eliminate Ψ and Φ from Eqs.(29,30) and, thus, obtain a pair of differential equations with respect to the perturbations δb , δA and $\delta \sigma$.

Since linearity in t has been assumed, the perturbations can appropriately be expressed as follows:

$$\delta b(y, t) = k_b(y) + h_b(y)t, \quad (45)$$

$$\delta A(y, t) = k_A(y) + h_A(y)t, \quad (46)$$

$$\delta \sigma(y, t) = k_\sigma(y) + h_\sigma(y)t. \quad (47)$$

The above forms allow us to re-express Eqs.(29,30) in the following manner:

$$k'_A - A'_o k_b - \frac{A''_o}{f_o} k_\sigma = C e^{12A_o}, \quad (48)$$

$$f_o k_b - k'_\sigma - 4A'_o k_\sigma = 4C f_o \frac{A'_o}{A''_o} e^{12A_o}, \quad (49)$$

$$h'_A - A'_o h_b - \frac{A''_o}{f_o} h_\sigma = 0, \quad (50)$$

$$f_o h_b - h'_\sigma - 4A'_o h_\sigma = 0, \quad (51)$$

where we have also made use of Eqs.(31,36,43,44).

Since the interest of this paper is on the possibility of time-dependent perturbations, we will restrict considerations to the last pair of differential equations (50,51). What is of issue is the existence of solutions that also satisfy Z_2 symmetry and the jump condition at the brane. By way of Eq.(28), these boundary conditions translate to:

$$h'_A(y = 0^+) = A'_o(y = 0^+) h_b(y = 0^+). \quad (52)$$

It is clear that an appropriate solution can be obtained when $h_b = h_\sigma = h'_A = 0$. That is:

$$\delta b = \delta \sigma = 0, \quad (53)$$

$$\delta A = h t, \quad (54)$$

where h is a constant. This solution describes gravitational perturbations in complete analogy with those identified by Binetruy et al. [20] (with regard to the ADKS-KSS model [13, 14]) and Diemand et al. [21] (with regard to the ADKS-KSS and KT [22] models). As discussed in these references, such time-dependent perturbed modes lead to dynamic instabilities in the otherwise static brane world.

4 Conclusion

In the preceding paper, we began the analysis by reviewing a self-tuning brane model that had recently been proposed by Kim, Kyae and Lee [1, 2]. A formulation of the static solution revealed how 4-dimensional Poincare invariance can be maintained without a fine-tuning of the external parameters. Unlike similarly proposed self-tuning scenarios, the KKL static solution has no awkward singularities to be dealt with.

The analysis continued with an investigation into the dynamical behavior of the KKL model. In particular, we linearized the relevant field equations about the static solution and then considered first-order perturbations for which the brane remains flat. It was shown that the complete first-order system (field equations and jump condition) supports a strictly gravitational perturbation that is linear in time. Such a perturbed mode is known to induce dynamical instability in the brane world scenario [20, 21]. Other detrimental consequences include a time-dependent Planck mass, an inevitable singularity in either the past or future, and a violation in energy conservation as seen by an observer on the brane [20].

To express the problem from a different perspective, the static solution can be viewed as a special member of a family of flat-brane solutions; these being parametrized by h , where $A(y, t) = A_o(y) + ht$ is the warp function. Since there is no known mechanism by which one can set $h = 0$ *a priori*, it is necessary, after all, to fine tune an external parameter in the KKL model.

Once again, in the context of the cosmological constant problem, an apparent self-tuning solution has been thwarted. In spite of this outcome, the KKL model is an intriguing approach to the cosmological constant problem and deserves further investigation.

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